Department of Engineering Science
Solid Mechanics and Materials Engineering

RIEG Rubber in Engineering Group

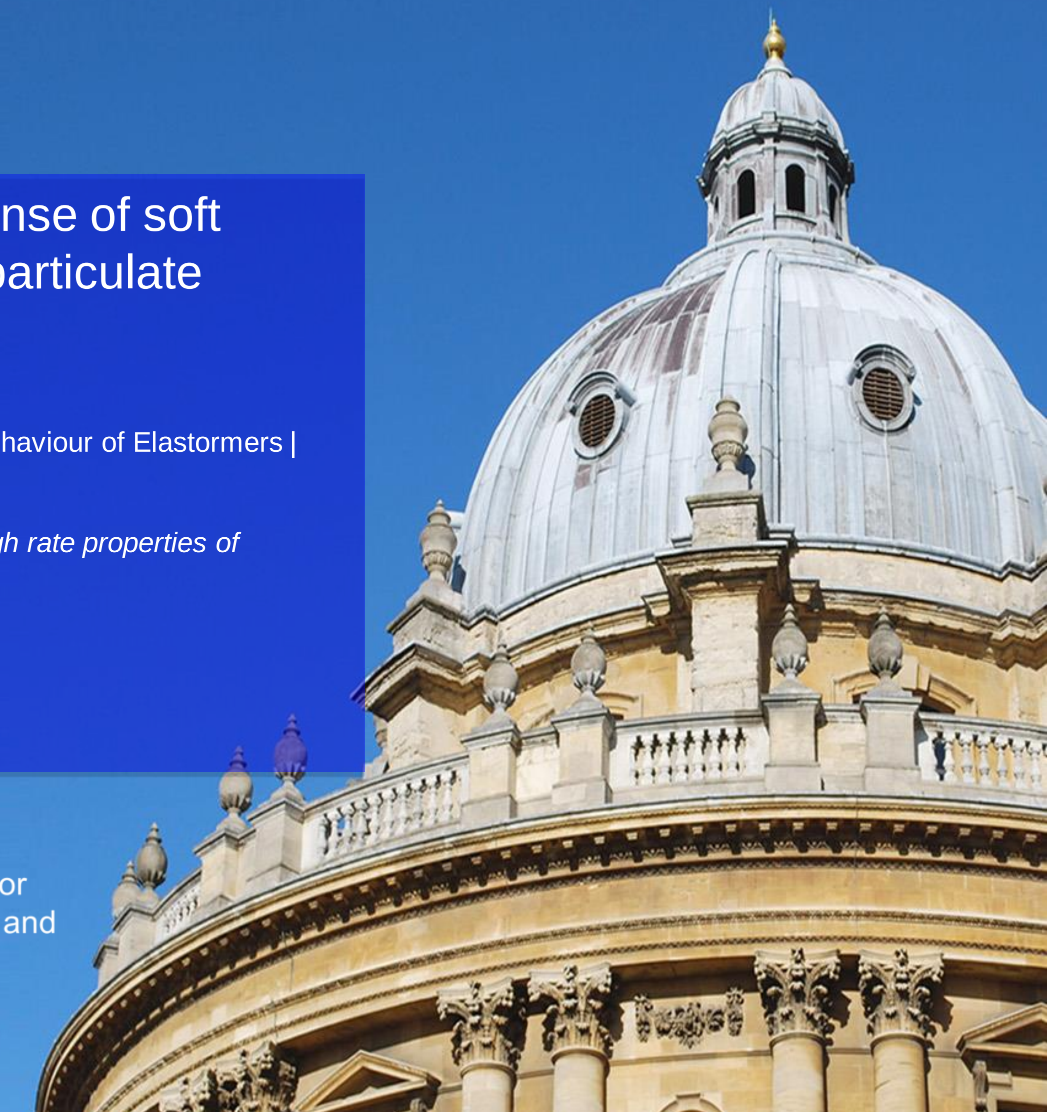
UNIVERSITY OF OXFORD

Predicting the high rate response of soft materials: From polymers to particulate composites

Rubber in Engineering Group: High Strain Rate Behaviour of Elastormers | Pembroke College, Oxford | 13 March 2020

Research conducted as part of a D.Phil. on the *High rate properties of particulate composites* at the University of Oxford.

Akash Trivedi
Supervisor: Prof. Clive Siviour



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Aim: To obtain the mechanical properties of soft polymers and their composites at high strain rates using simple, reliable, quasi-static experiments.

Why? Conventional techniques for high strain rate experimentation for soft materials do not give accurate measurements due to experimental artefacts.

How?

Experimental →  ← Constitutive Modelling

DMA: Dynamic Mechanical Analysis

TTS: time-temperature superposition

QS: Quasi-static

Get polymer → Use DMA and the TTS technique to obtain master curve/shift factors → Combine with QS experimental data to obtain relevant model parameters → Predict the material response at any suitable rate

Neoprene rubber test material to develop initial modelling framework [1,2]

↓

Plasticised PVC from a previous study [3] to refine the framework [4]

[1] Trivedi, A.R. & Siviour, C.R., (Accepted). A simple rate and temperature dependent hyperelastic model applied to capture the high strain rate behaviour of neoprene rubber. Dynamic Behaviour of Materials.
[2] Trivedi, A.R. & Siviour, C.R., 2018. *Framework for analyzing hyper-viscoelastic polymers.* In AIP Conference Proceedings
[3] Kendall, M.J. & Siviour, C.R., 2014. *Rate dependence of poly(vinyl chloride), the effects of plasticizer and time-temperature superposition.* Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences.
[4] Trivedi, A.R. & Siviour, C.R., (Accepted). *A novel methodology for predicting the high rate mechanical response of polymers from low rate data: Application to (plasticised) poly(vinyl chloride).* Mechanics of Time-Dependent Materials.

Plasticised and unplasticised PVC

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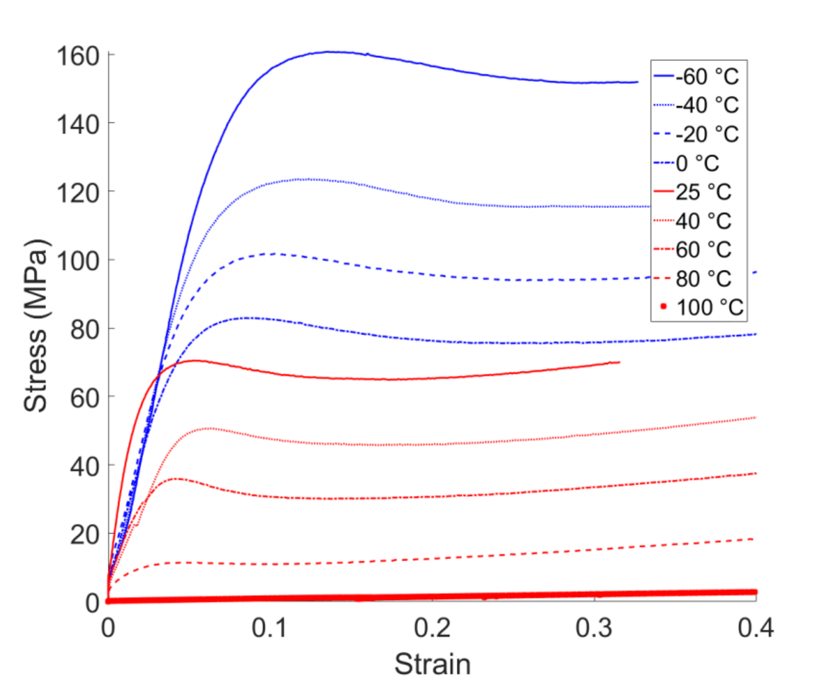
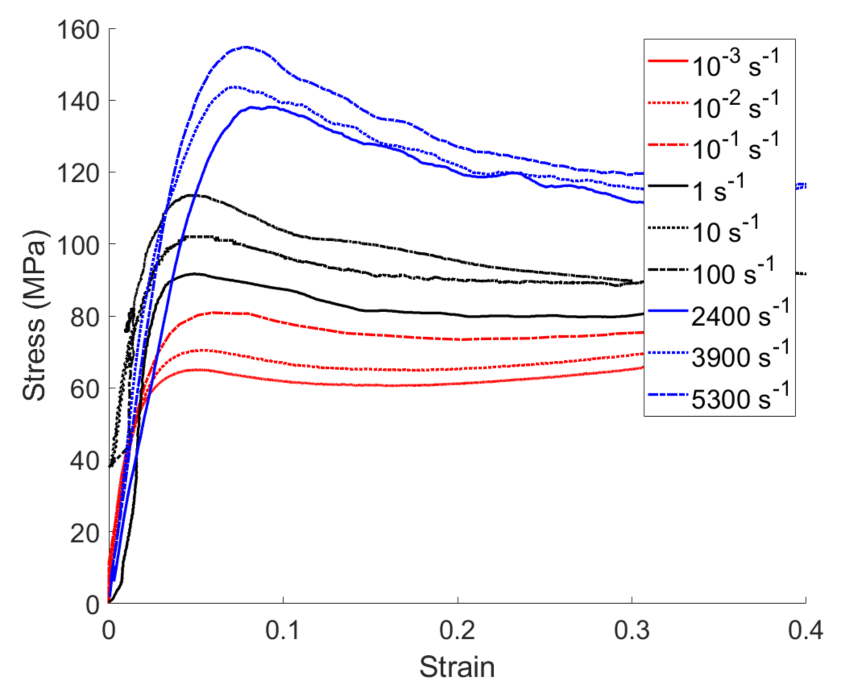
Rate-temperature equivalence

Results of varying temperature tests



Results of varying rate tests



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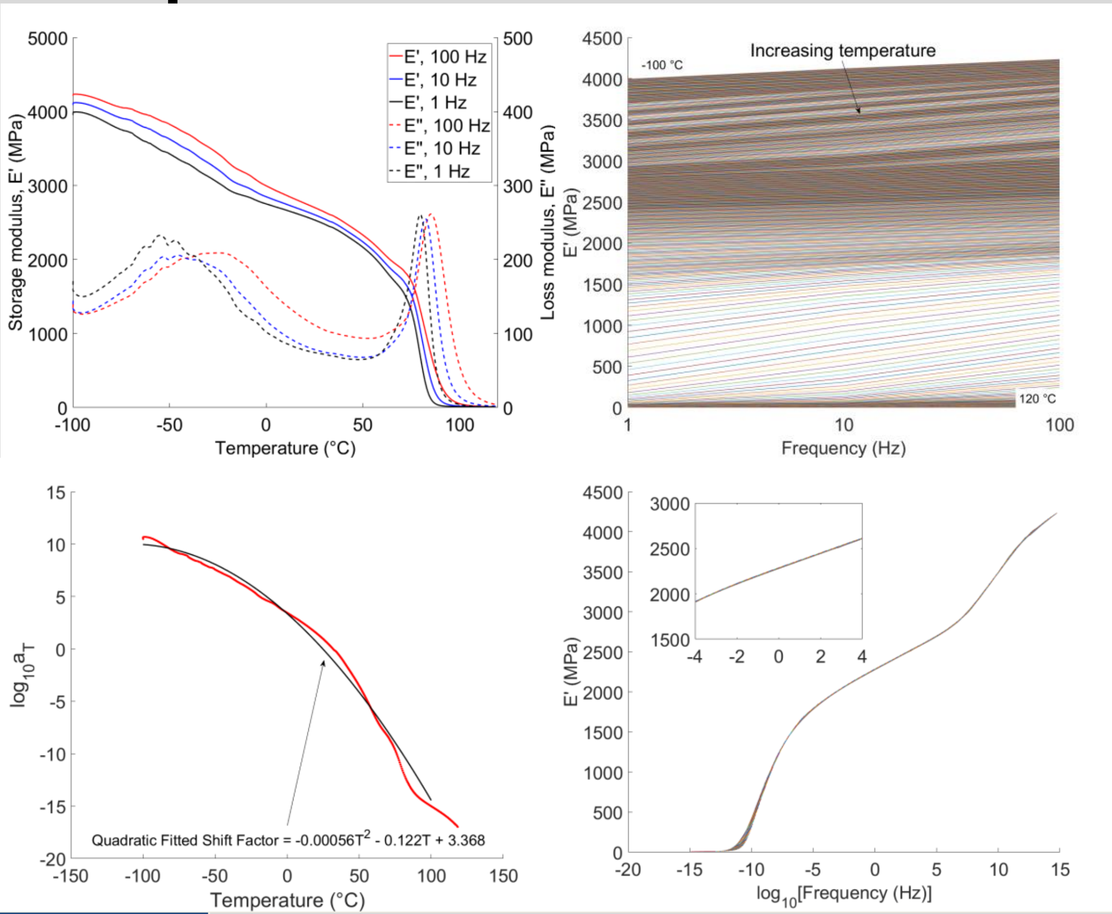
DMA experiments

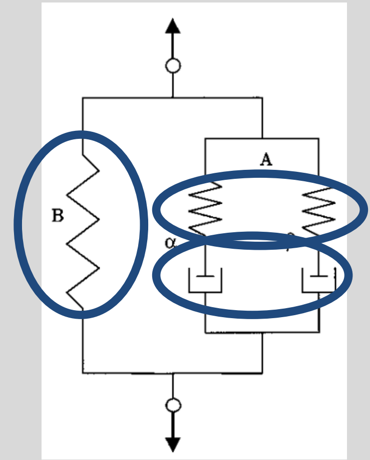
- Dual cantilever test from -100 °C to 120 °C
- Frequency sweep of 1, 10, 100 Hz
- Rectangular sample with dimensions 60 x 10 x 5 mm
- Master curve produced by shifting isotherms left or right in relation to the reference temperature of 25 °C
- Quadratic shift factor relationship observed



Modelling framework

Needed:
- Hyperelasticity for large strain behaviour
- Viscoplasticity for rate dependent plasticity
- Viscoelasticity for rate dependent elasticity
- Effects of adiabatic heating and subsequent temperature rise leading to thermal softening

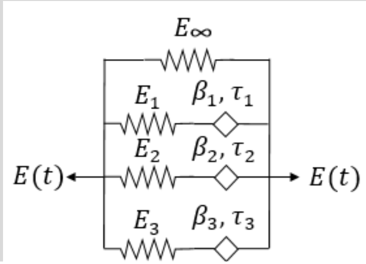
Delivered by:
- Langevin chain statistics
- Mulliken-Boyce [5] model basis
- FD model fit to the DMA experiments
- Viscoelastic modulus changed based on shifts derived from temperature rise



[5] Mulliken, A.D. & Boyce, M.C., 2006. Mechanics of the rate-dependent elastic–plastic deformation of glassy polymers from low to high strain rates. *International Journal of Solids and Structures*

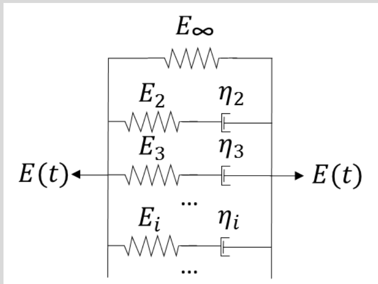
Fractional Derivative (FD) model

10-term fractional SLS model

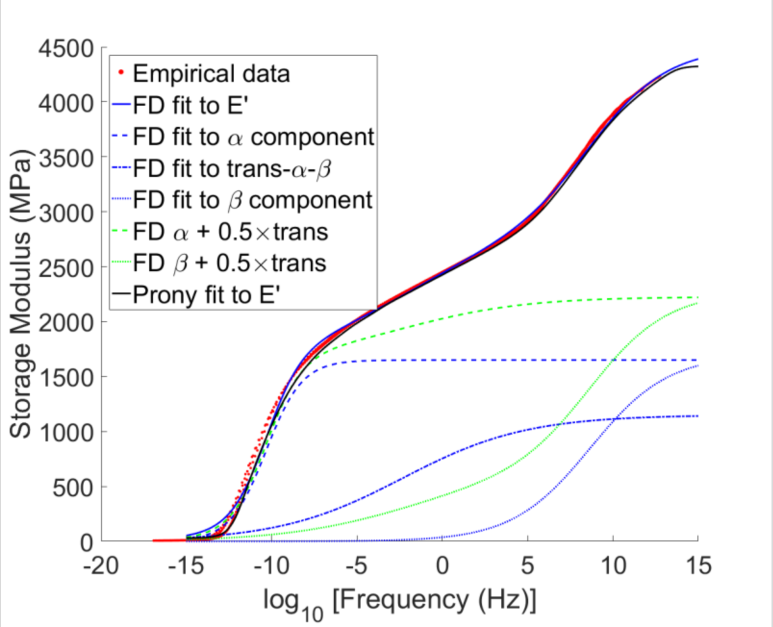


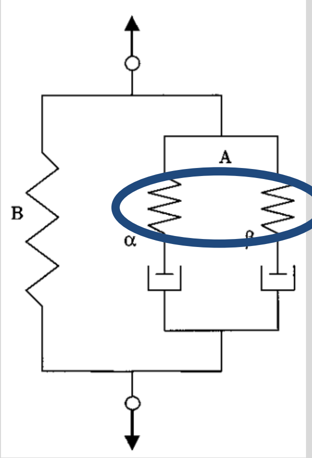
$$E^* = E' + iE'' = E_\infty + \sum_{i=1}^{M}\left[E_i \frac{(if)^{\beta_i}}{(if)^{\beta_i} + t_i^{-\beta_i}}\right]$$

26-term Prony SLS model



$$E^* = E' + iE'' = E_\infty + \sum_{i=1}^{N}\left[E_i \frac{if}{if + t_i^{-1}}\right]$$

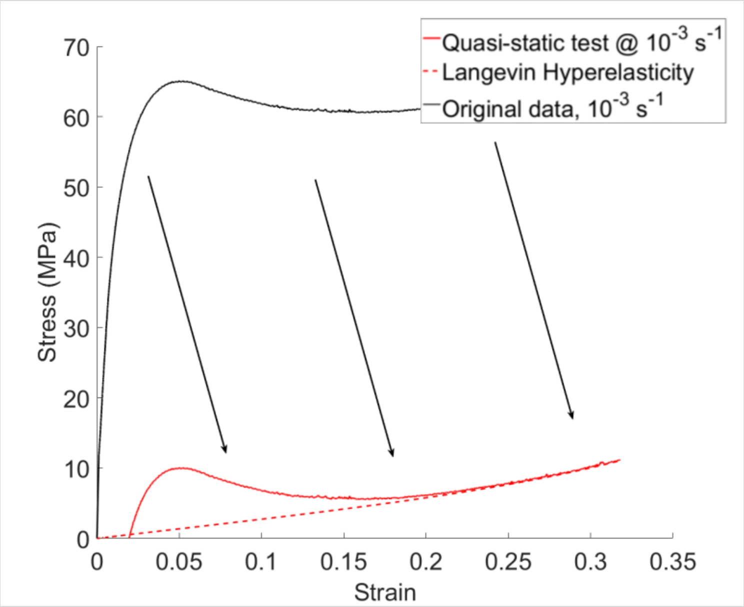


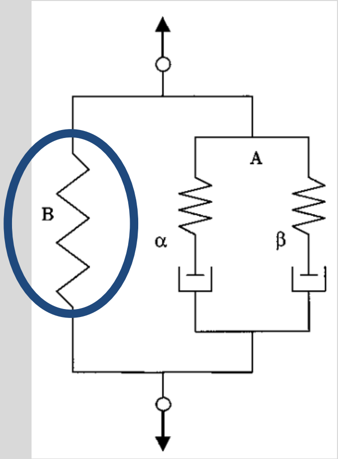


Prony series (MPa)	E_∞	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
	30.39	113.9	114.9	127.7	160.7	207.3	210.8	209.1	191.1	138.2
	E_{10}	E_{11}	E_{12}	E_{13}	E_{14}	E_{15}	E_{16}	E_{17}	E_{18}	E_{19}
	107.9	90.19	82.97	81.04	81.8	80.81	86.94	72.88	110.2	82.12
	E_{20}	E_{21}	E_{22}	E_{23}	E_{24}	E_{25}				
	127.0	178.0	219.3	347.9	362.5	598.6				
Fractional model	E_∞ (MPa)	E_1 (MPa)	β_1	t_1 (s)	E_2 (MPa)	β_2	t_2 (s)	E_3 (MPa)	β_3	t_3 (s)
	3	1650	0.41	4×10^7	1150	0.12	4×10^{-1}	1700	0.19	4×10^{-12}

Modelling results: Langevin

- Two parameter Langevin hyperelasticity
- Fit to quasi-static compression test





$$\mathcal{L}(\beta) \equiv \coth(\beta) - \frac{1}{\beta}$$

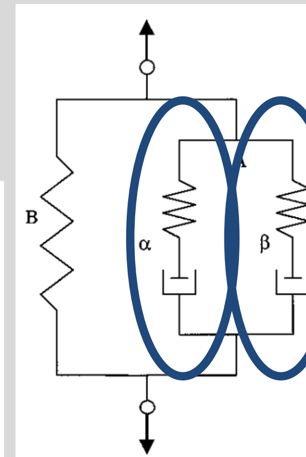
$$\lambda_{chain}^{p} = \sqrt{\frac{1}{3}\left(\varepsilon_n^2 + \frac{2}{\varepsilon_n}\right)}$$

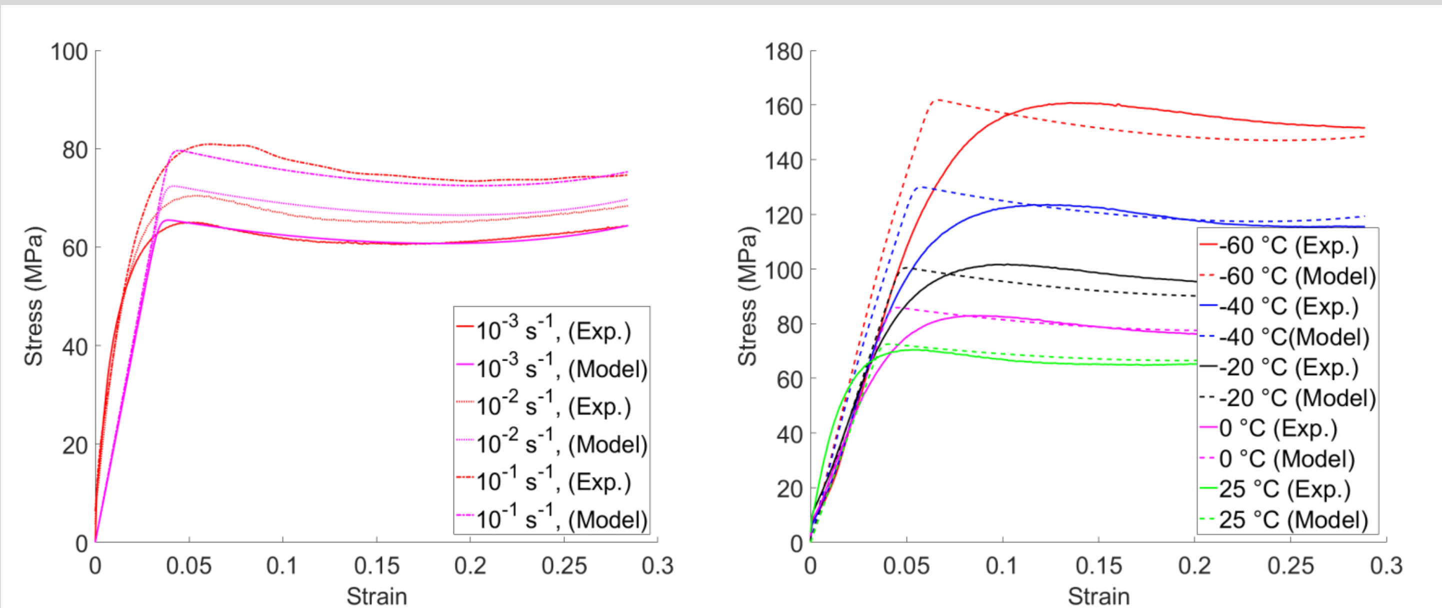
$$\sigma_L = \frac{C_R}{3}\frac{\sqrt{N}}{\lambda_{chain}^{p}}\mathcal{L}^{-1}\left(\frac{\lambda_{chain}^{p}}{\sqrt{N}}\right)(\varepsilon_n^2 - \varepsilon_n^{-1})$$

C_R, rubbery modulus
$\sqrt{N}$, limiting chain extensibility
ε_n, nominal strain

Modelling results: Alpha + Beta

- Alpha parameters are fit only to low rate data
- Beta parameters are fit to low temperature data





Time-Temperature Superposition principle is key to this approach

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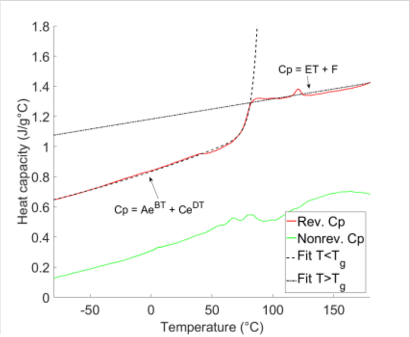
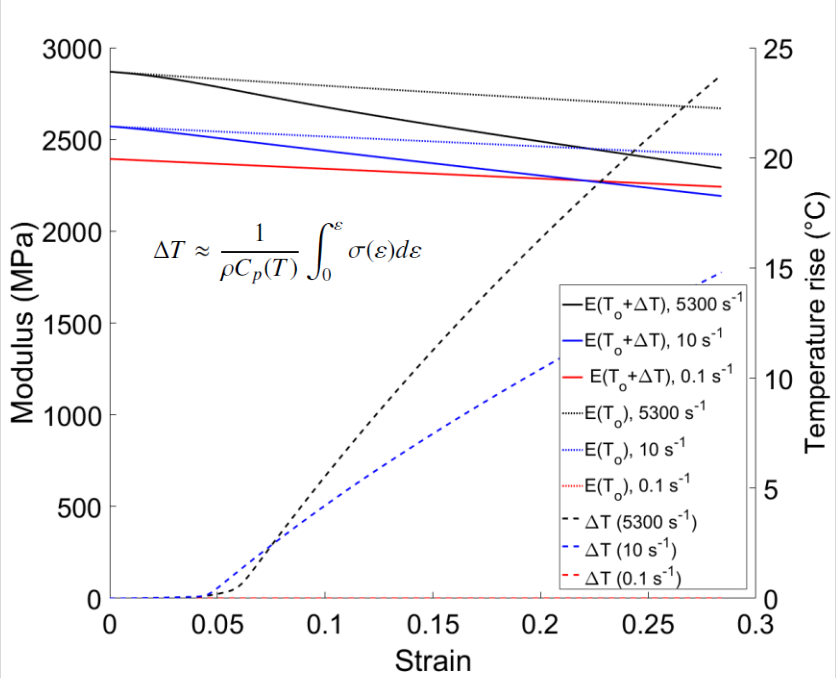
Adiabatic effects



- At higher rates, compression transitions from isothermal to adiabatic
- Two fits either side of the Tg on the DSC results were used to approximate the heat capacity of the PVC
- All mechanical work assumed to be converted to heat; temperature rise calculated assuming adiabatic process
- The temperature rise leads to thermal softening of the modulus as shown



DSC: Differential Scanning Calorimetry

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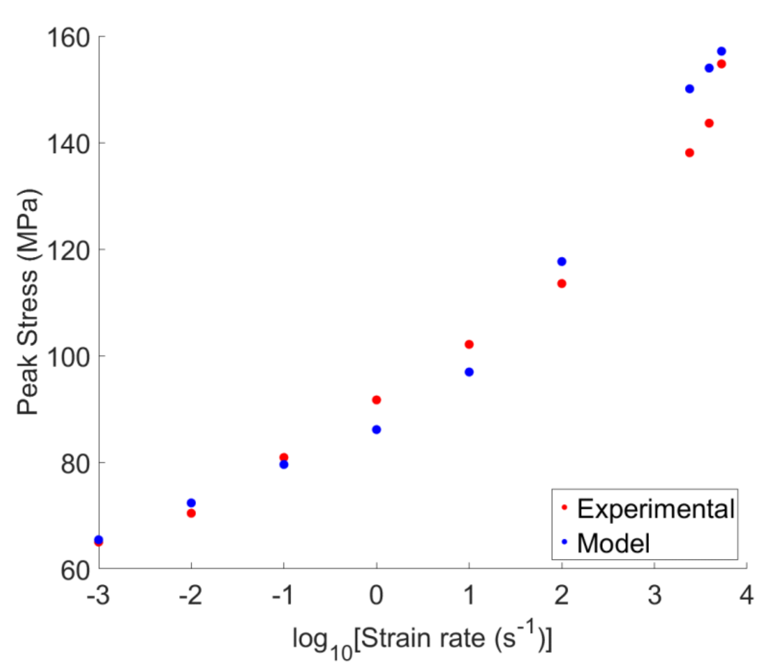
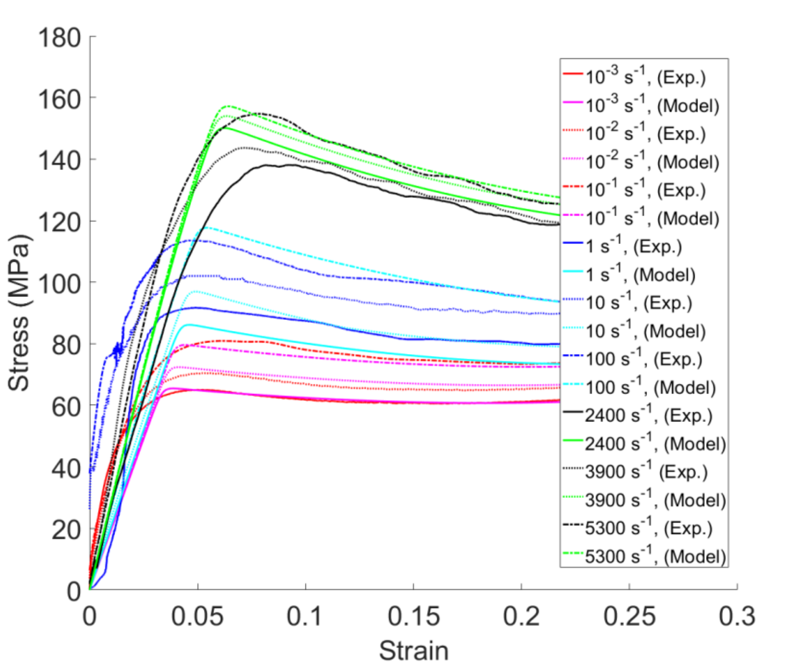
High rate prediction and validation





Unfilled and glass microsphere filled natural rubbers composites

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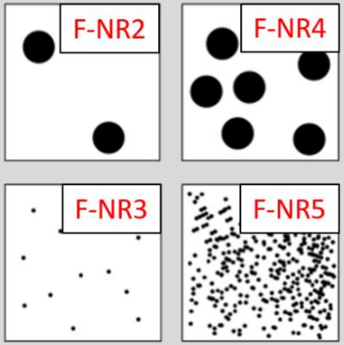
Particulate composites

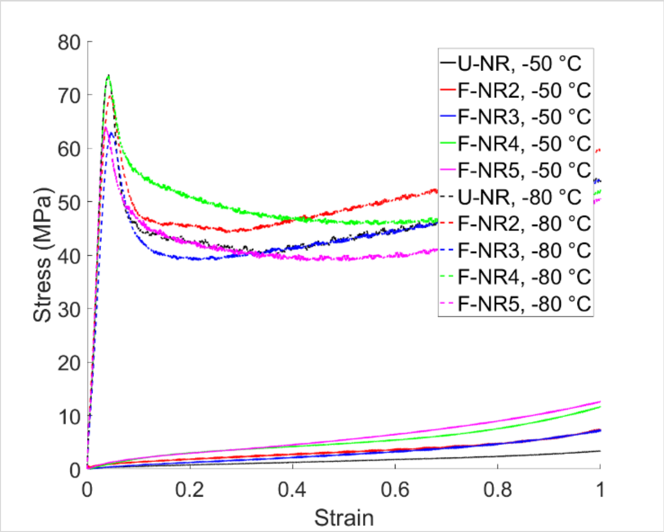
5% 50%

~100 µm

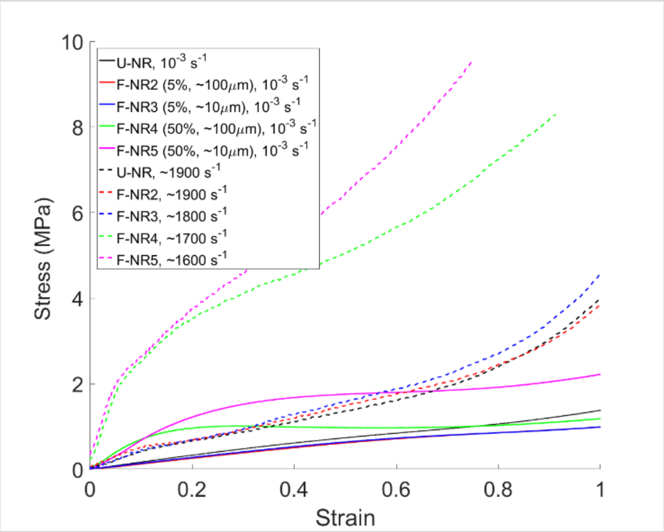
~10 µm



Varying temperature tests

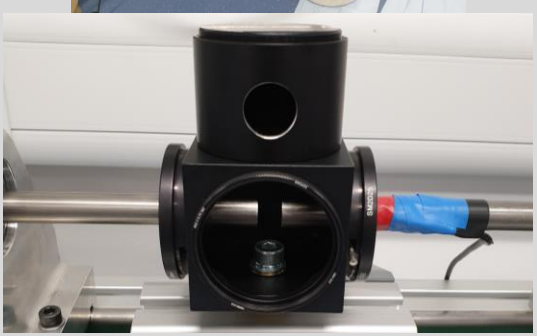


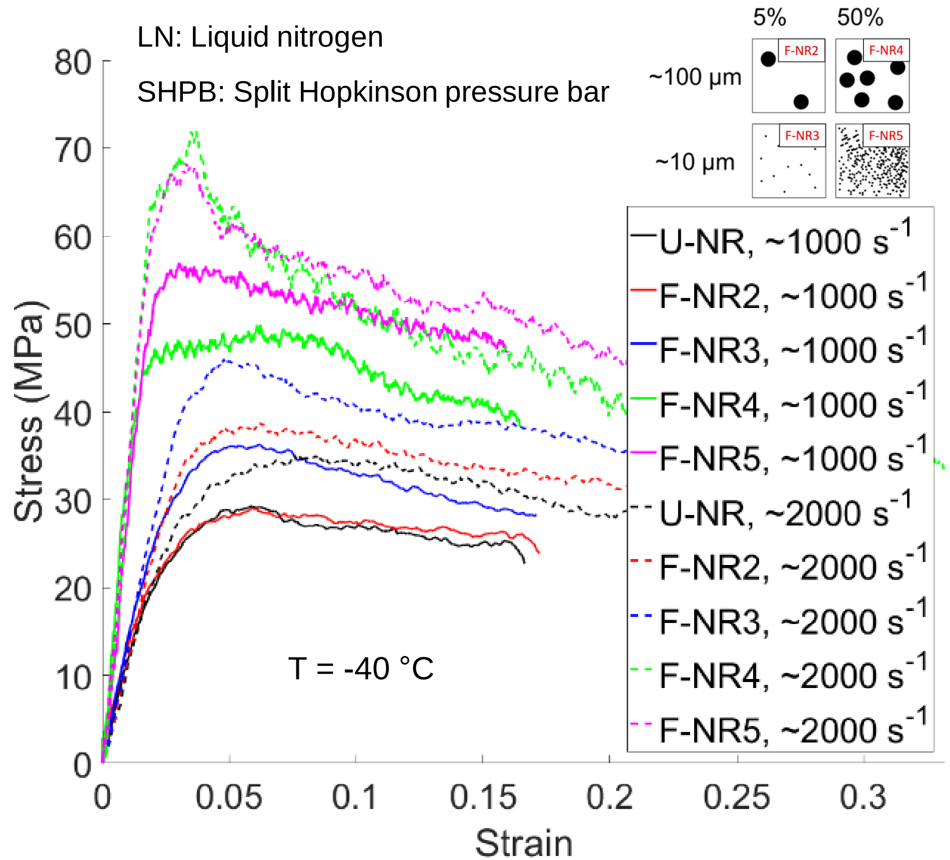
Varying rate tests



Experimental | Constitutive Modelling

LN Immersion Chiller for SHPB





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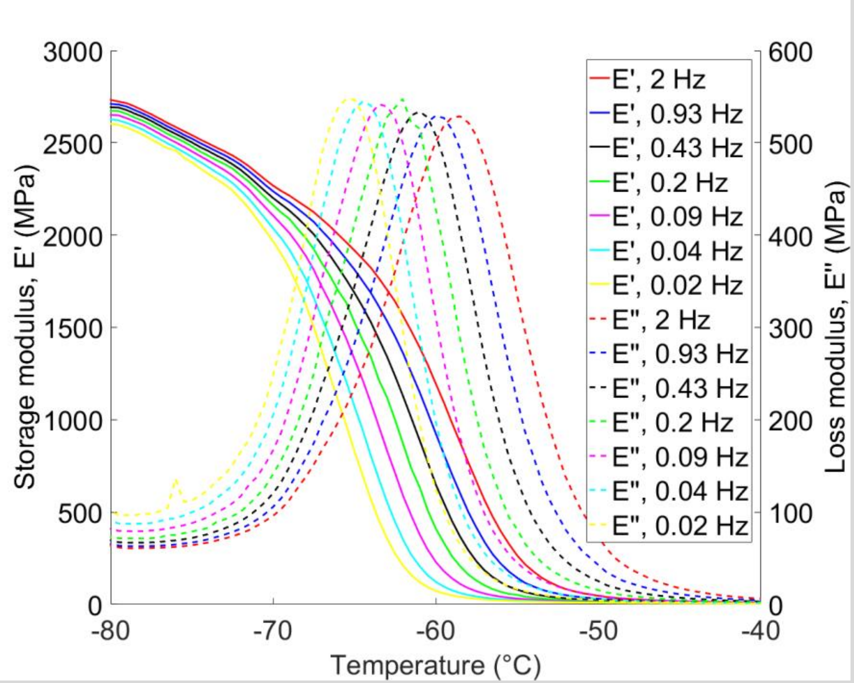
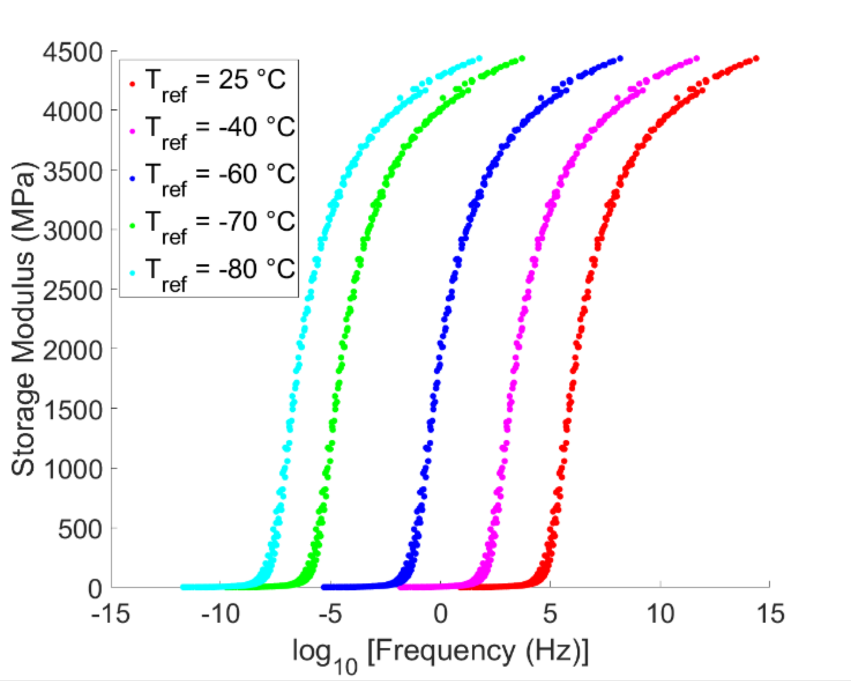
TTS Based Modelling Framework

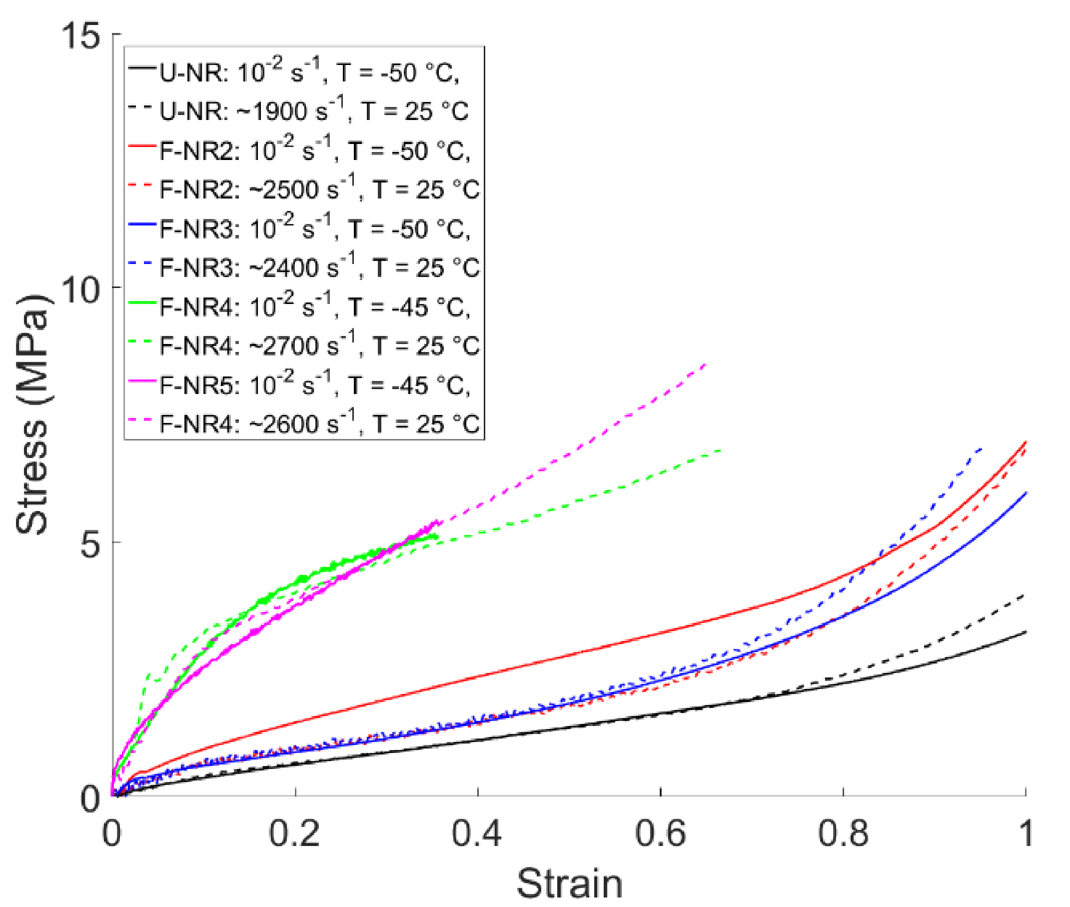




Experimental

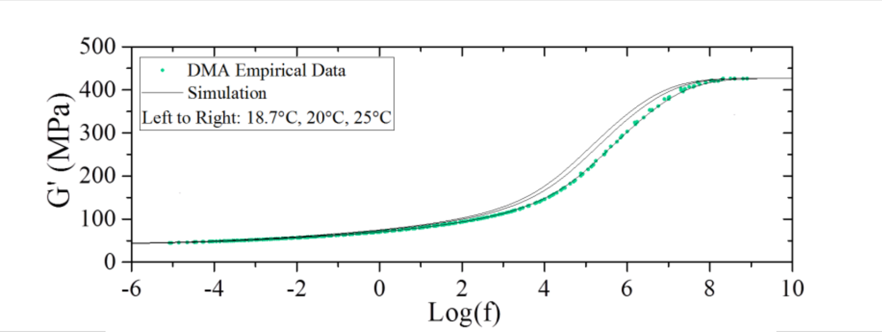
Constitutive Modelling

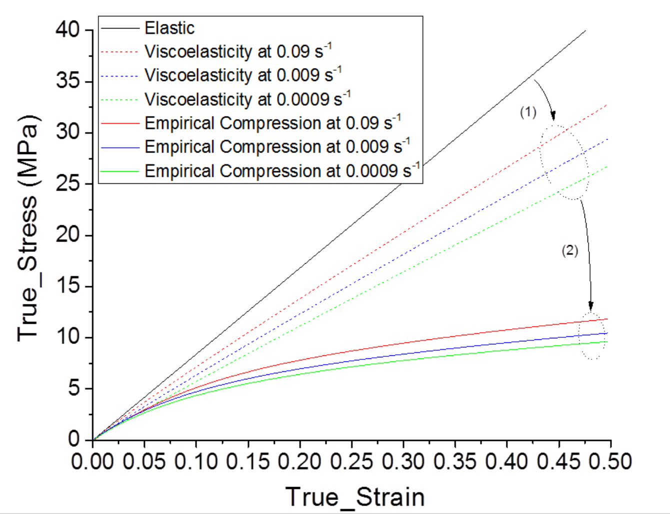
TTS Based Modelling Framework



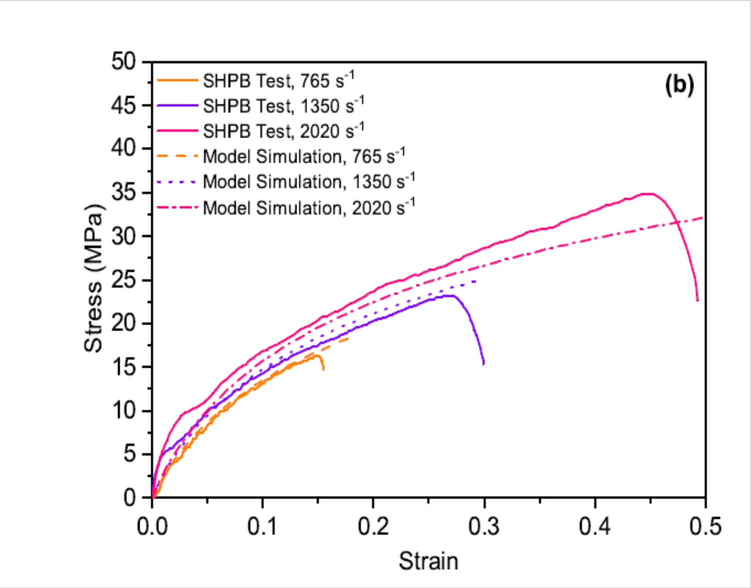
Viscoelastic Damage

Calibration to DMA and low rate data





Prediction of other responses



H. Chen
(Experimental Mechanics, Accepted)

Challenges

- Experimental artefacts for low-impedance materials at high strain rates
- Rate and/or temperature driven structural evolution
- TTS requires thermo-rheologically simple materials

Opportunities

- Novel technique development using full-field imaging and analysis
- Models based on calorimetry, microscopy and tomography
- Collaborations to approach problem from different angles

Keeps us engaged, employed and funded!

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Thank you for listening

Any questions?

Take a **picture** for **contact details** and **more research**!

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